DYNAMICS OF POPULATION AND RESOURCES

VERY SHORT SUMMARY OF THE BOOK BY MARIO LUDOVICO
PUBLISHED WITH THE TITLE "DINAMICA DI POPOLAZIONE E RISORSE"
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1. Introduction and synthesis

There is - since a very long time - a human need for satisfactory explanations for the natural laws that govern the appearance, growth and (sometimes) disappearance of biological populations, particularly concerning human populations. At variance with possible expectations, there is not an easy answer to such a licit curiosity.

The answer given in the 18th century by Malthus[1], with his exponential growth law for human population, has been taken for good up to a relatively recent time, notwithstanding the reasonable conceptual correction introduced by Verhulst[2] a few decades later.

Nowadays, the analysis of population dynamics has become a field of highly sophisticated theoretical exercises, though most of the visible results leave much to desire. Demographers have still to tackle serious problems, when requested to deliver reliable projections.

The contents of the book regarded by this summary is also one of these exercises, thought the degree of sophistication has been minimised, with a view to providing more manageable analytical instruments. In this connection, it is worth pointing out that the model of ecosystem presented here has no pretence to scientific work. The aim is only to introduce a relatively simple and clear logical framework in analysing the relationships between populations and the resources used for survival and development purposes.

The finding of this analysis leads to the following two major conclusions:

- *(i)* no stable equilibrium between any population and its environment is in general to be expected:
- *(ii)* any population, whose number of components does not decline with time, tends to extend constantly the mass of its resources along with the ambit of its ecosystem.

2. The basic logical framework of the model

With respect to any given population, the model represents the relevant ecosystem as a set of three major sectors that interact with each other as well as with themselves.

The ecosystem's components are grouped according to the following sectors:

(1) *Population* (i.e., the study population)
(2) *Resources* proper to the population
(3) *Environment*, i.e., all the remaining part of the ecosystem that doesn't belong either to the population or to its resources.

The interactions between these three sectors, and inside each of them, consist of specific flows of biological and non-biological materials, *and* energy in various forms.

The *Environment* is the fundamental source of raw materials that go to form the specific *Resources.* The *Resources* "feed" the *Population*, which, in its turn, directly and/or indirectly, returns

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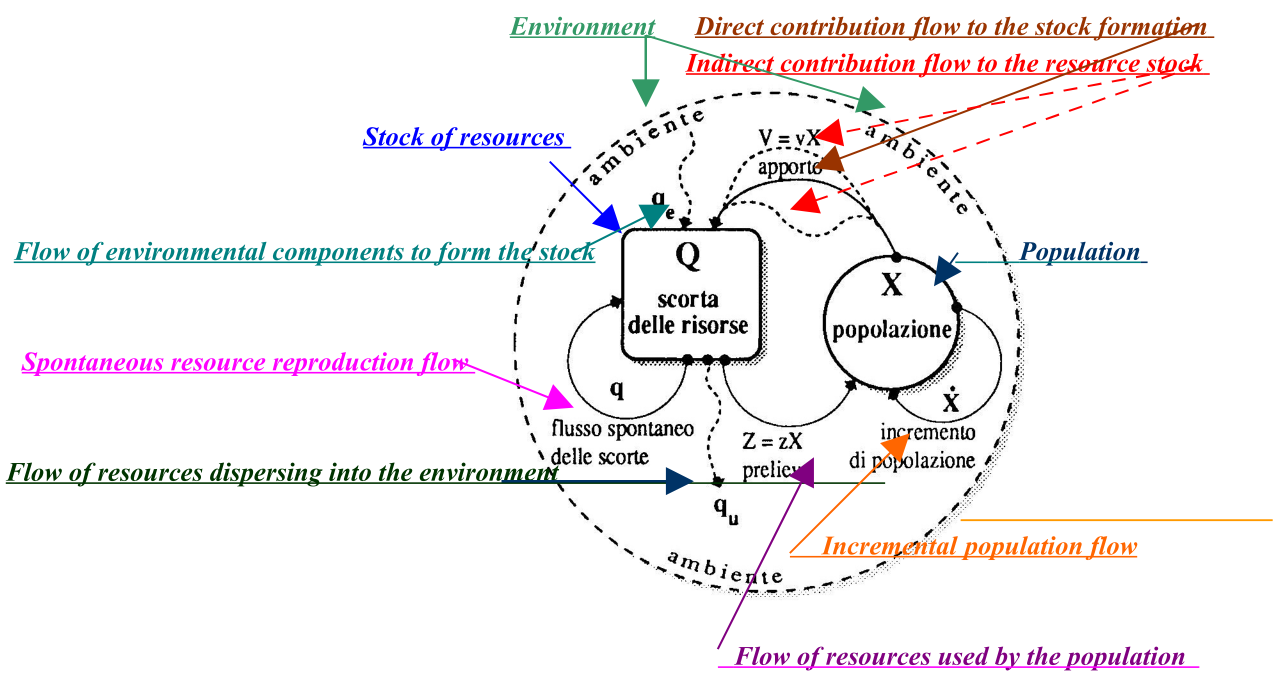
[1] Thomas Robert Malthus (1766-1834), *Essay on the Principle of Populations as it Affects the Future Improvement of Society,* London 1798

[2] Pierre François Verhulst (1804-1846), Belgian Mathematicians, formulated the so-called *logistic function*, to express the concept that population growth is constrained by environmental resources, first of all by the limited amount of physical space (territory) pertaining to each particular population. It is interesting to remark that the work by Verhulst remained actually ignored for nearly a century.

part of discarded materials to the *Environment*, while, as it is for example with human populations, does also "feed" its own stock of resources through the transformation of materials taken from its *Resources*. A schematic representation of this ecosystem is given by the figure below.

A GRAPHIC SCHEME OF THE MODELED ECOSYSTEM RELEVANT TO ANY GIVEN BIO-POPULATION



In this figure, "flow" means "amount of materials, energies, resources, population, etc. that transfers from one sector of the ecosystem to another (or to the sector itself) in a given time unit".

2.1 The basic hypotheses

The model logic framework results from the processing of three hypotheses (postulates):

(I) Variations in the resource flow dQ/dt, of the resources Q absorbed by the population mass X, can only be either a linear function of population increment dX, or a linear function of the acceleration in the incremental population flow dX/dt: i.e.,

either $d(dQ/dt) = \lambda\, dX$

or $d(dQ/dt) = \mu\, d(dX/dt)$

in which λ and μ are two coefficients of proportionality, and t indicates time;

(II) The flow of spontaneous (autonomous) resource regeneration $q = dq^*/dt$ varies with time according to a periodic trend, i.e.:

$$q = d\,q^*/dt = q_1\cos^2 ut - q_2\sin^2(ut + \varphi/2).$$

In this equation, q^* represents the mass of those materials and energies that belong to the sector of *Resources Q* but which are not absorbed by *Population X,* since they remain absorbed by the resource sector itself for self-reproduction ends; q_1, q_2, u and φ are positive constants;

(III) The flow Z of resources absorbed by the population is the following function of the population mass X:

$$Z = (s + \sigma y)X,$$

in which y is the population reproduction rate at instant t, s is the average resource consumption per population unit when $y = 0$, and σ is the increment (positive or negative) in the average resource consumption per population unit when $y \neq 0$.

In parallel to the function above, the direct or indirect (through mechanisms proper to the environment) contribution V provided by the population to the regeneration of its resources is expressed by

$$V = (r + \rho y)X,$$

in which r is the average rate of "restitution" per population unit when $y = 0$, and ρ is the increment (positive or negative) in the average restitution rate, per population unit, if $y \neq 0$.

3. The model equations

The postulates above lead to the interesting conclusion that the dynamics of the ecosystem may be summarised and described by a simple linear differential equation of the following form

$$\alpha dX/dt + bX + k\cos(wt+\psi) + g = 0,$$

in which α, b, k, w, ψ, and g are real constants.

From the linear differential equation above, and through a series of other relationships and definitions, one obtains different functions that express the possible evolution of both population X and resources Q, according to the different values that can be established for the equation constants.

In particular, for population X the following evolution equations can be written:

$X = A + B\,e^{-ct} + C\sin(wt+2\psi)$,
if $\alpha \neq 0$ and $b \neq 0$, or

$X' = A' + B't + C'\sin(wt + \psi)$,
if $\alpha \neq 0$ and $b = 0$, or

$X'' = A'' + C''\cos(wt + \psi)$,
if $\alpha = 0$ and $b \neq 0$.

In the preceding equations, A, A', A'', B, B', B'', C, C', C'', and ψ are integration constants that depend on the border conditions. Real constant c in the exponential is not nil.

In correspondence with these population dynamics equations, the following equations can be written for the dynamics of resources Q, i.e., respectively:

$Q = Pt + Ne^{-ct} + N\sin(wt + 2\psi) + R$,
if $\alpha \neq 0$ and $b \neq 0$, or

$Q' = P't^2 + M't + N'\sin(wt + \psi) + R'$,
if $\alpha \neq 0$ and $b = 0$, or

$Q'' = P''t + N''\cos(wt + \psi) + R''$,
if $\alpha = 0$ and $b \neq 0$.

Also in these equations, all the quantities that differ from independent variable t are integration constants, with $c \neq 0$ in the exponential.

It is not possible to justify here the conclusions mentioned in points *(i)* and *(ii)* of the introduction. The subject is instead addressed and discussed in the book text.